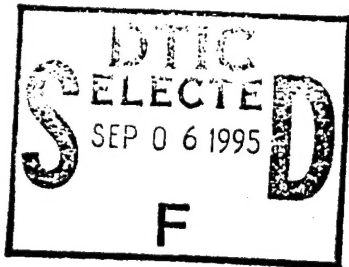


NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

REPORT 986

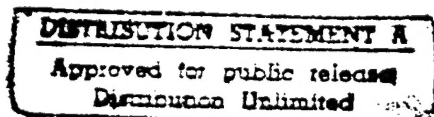
THE REVERSIBILITY THEOREM FOR THIN AIRFOILS  
IN SUBSONIC AND SUPERSONIC FLOW

By CLINTON E. BROWN



NAVY RESEARCH SECTION  
SCIENCE DIVISION  
REFERENCE DEPARTMENT  
LIBRARY OF CONGRESS

APR 6 1951



1950

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Yearly subscription, \$3.50; foreign, \$4.50; single copy price varies according to size . . . . . Price 15 cents

19950831 092

DTIC QUALITY INSPECTED 5

407813  
w7813

# AERONAUTIC SYMBOLS

## 1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbrevia- tion	Unit	Abbreviation
Length.....	$l$	meter.....	m	foot (or mile).....	ft (or mi)
Time.....	$t$	second.....	s	second (or hour).....	sec (or hr)
Force.....	$F$	weight of 1 kilogram.....	kg	weight of 1 pound.....	lb
Power.....	$P$	horsepower (metric).....		horsepower.....	hp
Speed.....	$V$	kilometers per hour.....	kph	miles per hour.....	mph
		meters per second.....	mps	feet per second.....	fps

## 2. GENERAL SYMBOLS

$W$	Weight = $mg$	$\nu$	Kinematic viscosity
$g$	Standard acceleration of gravity = $9.80665 \text{ m/s}^2$ or $32.1740 \text{ ft/sec}^2$	$\rho$	Density (mass per unit volume)
$m$	Mass = $\frac{W}{g}$		Standard density of dry air, $0.12497 \text{ kg-m}^{-3}$ at $15^\circ \text{ C}$ and $760 \text{ mm}$ ; or $0.002378 \text{ lb-ft}^{-3} \text{ sec}^2$
$I$	Moment of inertia = $mk^2$ . (Indicate axis of radius of gyration $k$ by proper subscript.)		Specific weight of "standard" air, $1.2255 \text{ kg/m}^3$ or $0.07651 \text{ lb/cu ft}$
$\mu$	Coefficient of viscosity		

## 3. AERODYNAMIC SYMBOLS

$S$	Area	$i_w$	Angle of setting of wings (relative to thrust line)
$S_w$	Area of wing	$i_t$	Angle of stabilizer setting (relative to thrust line)
$G$	Gap	$Q$	Resultant moment
$b$	Span	$\Omega$	Resultant angular velocity
$c$	Chord	$R$	Reynolds number, $\rho \frac{Vl}{\mu}$ where $l$ is a linear dimen- sion (e.g., for an airfoil of $1.0 \text{ ft}$ chord, $100$ mph, standard pressure at $15^\circ \text{ C}$ , the corre- sponding Reynolds number is $935,400$ ; or for an airfoil of $1.0 \text{ m}$ chord, $100 \text{ mps}$ , the corre- sponding Reynolds number is $6,865,000$ )
$A$	Aspect ratio, $\frac{b^2}{S}$	$\alpha$	Angle of attack
$V$	True air speed	$\epsilon$	Angle of downwash
$q$	Dynamic pressure, $\frac{1}{2} \rho V^2$	$\alpha_0$	Angle of attack, infinite aspect ratio
$L$	Lift, absolute coefficient $C_L = \frac{L}{qS}$	$\alpha_i$	Angle of attack, induced
$D$	Drag, absolute coefficient $C_D = \frac{D}{qS}$	$\alpha_a$	Angle of attack, absolute (measured from zero- lift position)
$D_0$	Profile drag, absolute coefficient $C_{D_0} = \frac{D_0}{qS}$	$\gamma$	Flight-path angle
$D_i$	Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$		
$D_p$	Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$		
$C$	Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$		

---

## REPORT 986

---

# THE REVERSIBILITY THEOREM FOR THIN AIRFOILS IN SUBSONIC AND SUPERSONIC FLOW

By CLINTON E. BROWN

Langley Aeronautical Laboratory  
Langley Field, Va.

---

1

Accession For	
NTIS CRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification .....	
By .....	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

# National Advisory Committee for Aeronautics

*Headquarters, 1724 F Street NW., Washington 25, D. C.*

Created by act of Congress approved March 3, 1915, for the supervision and direction of the scientific study of the problems of flight (U. S. Code, title 50, sec. 15). Its membership was increased from 12 to 15 by act approved March 2, 1929, and to 17 by act approved May 25, 1948. The members are appointed by the President, and serve as such without compensation.

JEROME C. HUNSAKER, Sc. D., Massachusetts Institute of Technology, *Chairman*

ALEXANDER WETMORE, Sc. D., Secretary, Smithsonian Institution, *Vice Chairman*

DETLEV W. BRONK, Ph. D., President, Johns Hopkins University.

JOHN H. CASSADY, Vice Admiral, United States Navy, Deputy Chief of Naval Operations.

EDWARD U. CONDON, Ph. D., Director, National Bureau of Standards.

HON. THOMAS W. S. DAVIS, Assistant Secretary of Commerce.

JAMES H. DOOLITTLE, Sc. D., Vice President, Shell Union Oil Corp.

R. M. HAZEN, B. S., Director of Engineering, Allison Division, General Motors Corp.

WILLIAM LITTLEWOOD, M. E., Vice President, Engineering, American Airlines, Inc.

THEODORE C. LONNQUEST, Rear Admiral, United States Navy, Deputy and Assistant Chief of the Bureau of Aeronautics.

DONALD L. PUTT, Major General, United States Air Force, Director of Research and Development, Office of the Chief of Staff, Matériel.

ARTHUR E. RAYMOND, Sc. D., Vice President, Engineering, Douglas Aircraft Co., Inc.

FRANCIS W. REICHELDERFER, Sc. D., Chief, United States Weather Bureau.

HON. DELOS W. RENTZEL, Administrator of Civil Aeronautics, Department of Commerce.

GORDON P. SAVILLE, Major General, United States Air Force, Deputy Chief of Staff—Development.

WILLIAM WEBSTER, M. S., Chairman, Research and Development Board, Department of Defense.

THEODORE P. WRIGHT, Sc. D., Vice President for Research, Cornell University.

HUGH L. DRYDEN, Ph. D., *Director*

JOHN W. CROWLEY, JR., B. S., *Associate Director for Research*

JOHN F. VICTORY, LL. D., *Executive Secretary*

E. H. CHAMBERLIN, *Executive Officer*

HENRY J. E. REID, D. Eng., Director, Langley Aeronautical Laboratory, Langley Field, Va.

SMITH J. DEFANCE, B. S., Director, Ames Aeronautical Laboratory, Moffett Field, Calif.

EDWARD R. SHARP, Sc. D., Director, Lewis Flight Propulsion Laboratory, Cleveland Airport, Cleveland, Ohio

## TECHNICAL COMMITTEES

AERODYNAMICS

POWER PLANTS FOR AIRCRAFT

AIRCRAFT CONSTRUCTION

OPERATING PROBLEMS

INDUSTRY CONSULTING

*Coordination of Research Needs of Military and Civil Aviation*

*Preparation of Research Programs*

*Allocation of Problems*

*Prevention of Duplication*

*Consideration of Inventions*

LANGLEY AERONAUTICAL LABORATORY,  
Langley Field, Va.

LEWIS FLIGHT PROPULSION LABORATORY,  
Cleveland Airport, Cleveland, Ohio

AMES AERONAUTICAL LABORATORY,  
Moffett Field, Calif.

*Conduct, under unified control, for all agencies, of scientific research on the fundamental problems of flight*

OFFICE OF AERONAUTICAL INTELLIGENCE,  
Washington, D. C.

*Collection, classification, compilation, and dissemination of scientific and technical information on aeronautics*

## REPORT 986

# THE REVERSIBILITY THEOREM FOR THIN AIRFOILS IN SUBSONIC AND SUPERSONIC FLOW

By CLINTON E. BROWN

### SUMMARY

*A method introduced by Munk is extended to prove that the lift-curve slope of thin wings in either subsonic or supersonic flow is the same when the direction of flight of the wing is reversed. It is also shown that the wing reversal does not change the thickness drag, damping-in-roll parameter  $C_{l_p}$ , or the damping-in-pitch parameter  $C_{m_q}$ .*

### INTRODUCTION

The present report makes use of and extends a paper by Munk (reference 1) in which simple dynamic concepts are used to prove that the lift-curve slope and thickness drag of supersonic airfoils with supersonic edges are the same when the airfoil is flown in a reversed direction. This extension of Munk's work provides a proof that the thickness drag, lift-curve slope, damping in roll, and the damping-in-pitch parameter  $C_{m_q}$  remain the same when any airfoil or system of airfoils is reversed in both subsonic and supersonic flow. The theorem applies to cases in which the trailing-edge velocities are finite; no restrictions are placed on plan form.

The reversibility theorem for drag was first obtained by different methods by Von Kármán (reference 2). Hayes has treated the lifting case for a restricted series of wing types at supersonic speeds. (See references 3 and 4.) Harmon (reference 5) has extensively treated the stability derivatives for a restricted group of plan forms at supersonic speeds.

### PROOF

Under the assumptions of the linearized potential-flow theory, it becomes possible to obtain a great simplification of subsonic and supersonic lifting-surface problems. The use of the linear equations of motion allows the boundary conditions on a lifting surface to be satisfied on a plane near the wing surface and permits the use of the superposition principle. Consider a set of Cartesian coordinates  $x, y, z$  in which the  $x$ -axis is taken in the flight direction and the  $z$ -axis, in the vertical direction. The boundary conditions become a stipulation of the vertical-velocity distribution over the projection of the wing surface on the  $xy$ -plane. As a result of this simplification, the effects of camber, twist, angle of attack, and thickness may be treated separately.

For the complete comprehension of the analysis to follow, it is necessary to understand the manner in which drag ultimately appears in the flow field. Two distinct forms of drag may be found: one associated with a trailing vortex system, the other with the production of waves. In the case

of a vortex wake, the drag shows up in the wake a great distance downstream in the form of a pressure defect which, when integrated over a plane normal to the flight path, yields the drag. This result is identical with that of incompressible flow. The drag produced by wave formation shows up in the field as a combined momentum and pressure defect; of course, the thin-airfoil theory predicts a wave drag only at supersonic speeds. In all cases, the total resistance may be obtained by integrating the momentum transport across the sides of a box enclosing the wing. It is often convenient to place the sides of the box at infinity and allow the top and bottom to approach the plane of the wing. This process yields for the drag

$$D = -2\rho \int \int \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial z} dy dx \quad (1)$$

where  $\rho$  is the stream density,  $\phi$  is the disturbance potential, and the integration taken over both upper and lower sides extends to infinity. Note that the drag is independent of the main stream direction but depends only on the disturbance potential  $\phi$ . In the usual problems, singularities occur on the wing leading edges and care must be taken with the integration if the quantities in the integrand of equation (1) are evaluated on the  $xy$ -plane. Neglect of the singular behavior leads to the omission of the leading-edge suction forces. For additional information on the fundamentals of the linear theory, see references 2 and 6.

**Thickness drag.** Consider a symmetrical airfoil at zero angle of attack. The potential of the flow may be expressed as

$$\Phi_1 = Vx + \phi_1 \quad (2)$$

where  $V$  is the stream velocity and  $\phi_1$  is the disturbance potential which satisfies the boundary condition

$$\frac{1}{V} \left( \frac{\partial \phi_1}{\partial z} \right)_{z=0} = \frac{dz}{dx} \quad (3)$$

where  $\frac{dz}{dx}$  is the airfoil surface slope. In addition, the potential must satisfy the usual conditions for physical flows such as the vanishing of the perturbation velocities at infinity for subsonic flow and undisturbed flow ahead of the foremost Mach waves in supersonic flow. Assume the main flow direction to be reversed. The new potential would result:

$$\Phi_2 = -Vx + \phi_2 \quad (4)$$

where  $\phi_2$  satisfies the condition

$$\frac{1}{V^2} \left( \frac{\partial \phi_2}{\partial z} \right)_{z=0} = -\frac{dz}{dx} \quad (5)$$

By superposing the solutions  $\Phi_1$  and  $\phi_2$  a new potential is formed:

$$\Phi_3 = \Phi_1 + \phi_2 \quad (6)$$

Such a step is quite permissible inasmuch as the differential equation governing the flows is independent of the sign of the stream velocity. The vertical velocity  $\left( \frac{\partial \Phi_3}{\partial z} \right)_{z=0}$  becomes zero and thus a boundary condition for a plate of zero thickness is satisfied. Inasmuch as there are no infinite induced velocities at the edges of the resultant wing and therefore no edge forces, the flat plate can produce no changes in stream momentum; hence, the momentum or pressure defects a great distance downstream in the flow must be equal to those upstream. Any momentum or pressure defects at infinity upstream arise from the reversed airfoil potential, and the momentum or pressure defects a great distance downstream arise only from the original airfoil potential. Since the drag of each airfoil is equal to the momentum or pressure defects in its wake, the drag of the two airfoils must necessarily be equal. It is well known that the drag of symmetrical bodies in subsonic potential flow is zero; hence, the reversibility of drag is most pertinent to supersonic flows. The preceding proof and discussion follows essentially that of Munk (reference 1).

**Lift-curve slope.**—Inasmuch as the lift-curve slope of a wing is independent of camber and twist, it is sufficient to treat a flat-plate airfoil at an angle of attack  $\alpha$ . Unlike the symmetrical-drag case, however, a certain indeterminacy exists in the potential whenever subsonic trailing edges are present. Subsonic edges occur when the component of stream velocity normal to the edge is subsonic. In order to remove this indeterminacy it is necessary to specify the circulation. The use of the Kutta condition is an appropriate means for this process because, in effect, an additional boundary condition is imposed. This requirement, that the velocities at the trailing edge be finite, is indeed a physical condition arising from the fact that the boundary layer, always present at trailing edges, would separate from the edge rather than accommodate the high adverse accelerations around the edge. It is exactly the Kutta condition which leads to unique solutions and which is necessary to prove the reversibility theorem.

The potential of the flat-plate airfoil may now be written:

$$\Phi_1 = Vx + \phi_1 \quad (7)$$

where  $\phi_1$  is the disturbance potential satisfying the conditions that the trailing-edge velocities are finite and the boundary condition

$$\frac{1}{V^2} \frac{\partial \phi_1}{\partial z} = -\alpha \quad (8)$$

The drag of the wing can be written:

$$D_1 = L_1 \alpha - F_1 \quad (9)$$

where  $F$  is the component parallel to the surface of the resultant force, usually known in linear theory as the leading-edge suction force, and  $L$  is the lift force.

As in the drag case, the reversed-stream velocity produces the potential

$$\Phi_2 = -Vx + \phi_2 \quad (10)$$

where  $\phi_2$  satisfies the Kutta condition and the boundary condition

$$\frac{1}{V^2} \frac{\partial \phi_2}{\partial z} = \alpha \quad (11)$$

The drag is now

$$D_2 = L_2 \alpha - F_2 \quad (12)$$

The superposition of the potential  $\phi_2$  on  $\Phi_1$  results in the flow over a flat plate of zero angle of attack.

The drag of the combined airfoils is now

$$D_3 = F_2 - F_1 \quad (13)$$

provided the superposition has not changed the leading-edge suction forces  $F_1$  and  $F_2$ . These suction forces have been shown (references 7 to 9) to be dependent on the asymptotic distribution of vorticity as the edge is approached; suction forces are obtained only when the vortex strength approaches infinity at the edge, this condition corresponding to infinite upwash velocity around the edge. Inasmuch as the superposition of a solution having finite-edge velocities does not alter the asymptotic strength of the singularities at the edge, it follows that the edge forces will be unchanged by the superposition.

When a momentum balance in the stream is formed, the upstream momentum and pressure defects in the combined-airfoil case differ from the downstream momentum and pressure defects by the difference in the suction forces  $D_3$ . The upstream momentum and pressure defects are, however, equal to  $D_2$ , whereas those downstream are equal to  $D_1$ . Therefore,

$$D_1 - D_2 = F_2 - F_1 \quad (14)$$

or from equations (9) and (12)

$$L_1 \alpha = L_2 \alpha \quad (15)$$

The lifts  $L_1$  and  $L_2$  are equal and, therefore, the lift-curve slopes are equal. Obviously, the lift-curve slopes of cambered and twisted wings are also unchanged when the airfoil is reversed. It is important to note that the drags are not equal unless the suction forces are zero or cancel.

In reference 4, the conclusion is reached that the lift theorem cannot be a general one; however, it appears that this conclusion was deduced from an equation of insufficient generality. Indeed, the analysis of the present report shows the lift theorem to apply to all plan forms so long as the Kutta condition is applied to subsonic trailing edges.

**Damping in roll.**—The proof for the reversibility of damping in roll proceeds in the same manner as that for the lift.

The rolling moment of the thin wing may be expressed as follows:

$$L_1' = \int_S \Delta p_1 y dS \quad (16)$$

where  $\Delta p$  is the pressure difference between the upper and lower surface and  $S$  is the wing area.

The drag of the linearly twisted wing used to represent the rolling flat plate is

$$D = \int_S \Delta p_1 \alpha dS - F_1 \quad (17)$$

The drag may be expressed as a function of the rolling moment inasmuch as  $\alpha = \frac{p y}{V}$  where  $p$  is the angular velocity in roll. For the twisted wing,

$$D_1 = \frac{p}{V} L_1' - F_1 \quad (18)$$

The drag of the reversed airfoil is then

$$D_2 = \frac{p}{V} L_2' - F_2 \quad (19)$$

Superposing the disturbance potential of the reversed airfoil again cancels the wing slopes, and the resulting momentum change at the combined airfoils becomes

$$D_3 = F_2 - F_1 \quad (20)$$

Establishing the conservation of momentum in the flow, as was done for the lifting case, gives the result:

$$\frac{p}{V} L_1' = \frac{p}{V} L_2' \quad (21)$$

Therefore, the rolling moment for the reversed airfoil is the same as that of the unreversed airfoil. It follows then that the rolling-moment derivative  $C_{lp}$  for any wing is unchanged by reversal.

**Steady pitching moment.**—The pitching moment of a wing undergoing a steady pitching velocity  $q$  about the point  $x_0$  may be written

$$M = \int_S (x - x_0) \Delta p dS \quad (22)$$

where  $x_0$  is the reference point about which moments are taken. The drag of the cambered-wing surface representing the steady pitching motion is

$$D = \int_S \Delta p \alpha dS - F \quad (23)$$

and the local angle of attack for such a wing is

$$\alpha = q \frac{x - x_0}{V} \quad (24)$$

Hence, the drag may be expressed from equations (22) to (24) as follows:

$$D = \frac{q}{V} M - F \quad (25)$$

Performing the superposition of reversed potential and original potentials yields an airfoil of zero angle of attack; the momentum balance, as for the steady rolling case, cancels the suction forces to leave:

$$\frac{q}{V} M_1 = \frac{q}{V} M_2 \quad (26)$$

The pitching moments of the two airfoils are the same and, therefore, the damping-in-pitch parameter  $C_{mq}$  is unchanged by a reversal of the wing.

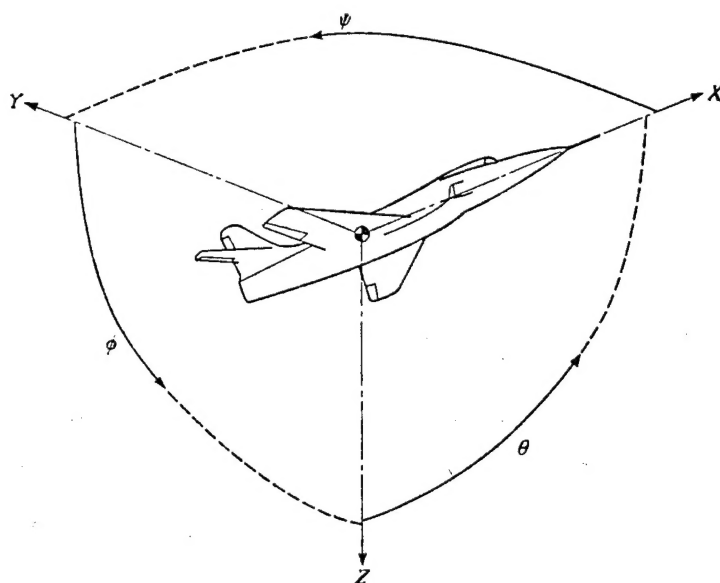
## DISCUSSION

Inasmuch as the analysis presented is unrestricted as to plan form, it follows that any system of airfoils will obey the reversibility theorem; this does not allow for the reversal of the individual airfoils but only for the reversal of the complete system. Indeed, the same result holds for groups of airfoils in different horizontal planes, provided the boundary conditions for each wing are satisfied in the plane of the wing. It should be noticed that the pitching-moment coefficients, lift coefficients due to pitching, and the constants arising from camber such as  $\alpha_{L=0}$  are not generally the same when the wing is reversed.

LANGLEY AERONAUTICAL LABORATORY,  
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,  
LANGLEY FIELD, VA., June 26, 1949.

## REFERENCES

1. Munk, M. M.: The Reversal Theorem of Linearized Supersonic Airfoil Theory. Naval Ordnance Lab. Memo. 9624, July 26, 1948.
2. Von Kármán, Theodore: Supersonic Aerodynamics—Principles and Applications. Jour. Aero. Sci., vol. 14, no. 7, July 1947, pp. 373–409.
3. Hayes, Wallace D.: Linearized Supersonic Flow. Rep. No. AL No. 222, North American Aviation Inc., June 18, 1947.
4. Hayes, Wallace D.: Reversed Flow Theorems in Supersonic Aerodynamics. Rep. No. AL-755, North American Aviation Inc., Aug. 20, 1948.
5. Harmon, Sidney M.: Theoretical Relations between the Stability Derivatives of a Wing in Direct and in Reverse Supersonic Flow. NACA TN 1943, 1949.
6. Prandtl, L.: Theorie des Flugzeugtragflügels im zusammendrückbaren Medium. Luftfahrtforschung, Bd. 13, Nr. 10, Oct. 12, 1936, pp. 313–319.
7. Von Kármán, Th., and Burgers, J. M.: General Aerodynamic Theory—Perfect Fluids. Theory of Airplane Wings of Infinite Span. Vol. II of Aerodynamic Theory, div. E, ch. II, sec. 10, W. F. Durand, ed., Julius Springer (Berlin), 1935, pp. 51 and 52.
8. Hayes, W. D., Browne, S. H., and Lew, R. J.: Linearized Theory of Conical Supersonic Flow with Application to Triangular Wings. Rep. No. NA-46-818, North American Aviation, Inc., Sept. 30, 1946. (Revised June 26, 1947), pp. 45–51.
9. Brown, Clinton E.: Theoretical Lift and Drag of Thin Triangular Wings at Supersonic Speeds. NACA Rep. 839, 1946.



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Sym- bol		Designation	Sym- bol	Positive direction	Designa- tion	Sym- bol	Linear (compo- nent along axis)	Angular
Longitudinal.....	X	X	Rolling.....	L	Y → Z	Roll.....	φ	u	p
Lateral.....	Y	Y	Pitching.....	M	Z → X	Pitch.....	θ	v	q
Normal.....	Z	Z	Yawing.....	N	X → Y	Yaw.....	ψ	w	r

Absolute coefficients of moment

$$C_l = \frac{L}{c b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q b S}$$

(rolling)                      (pitching)                      (yawing)

Angle of set of control surface (relative to neutral position),  $\delta$ . (Indicate surface by proper subscript.)

#### 4. PROPELLER SYMBOLS

$D$	Diameter	$P$	Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$
$p$	Geometric pitch	$C_s$	Speed-power coefficient $= \sqrt{\frac{\rho V^5}{P n^2}}$
$p/D$	Pitch ratio	$\eta$	Efficiency
$V'$	Inflow velocity	$n$	Revolutions per second, rps
$V_s$	Slipstream velocity	$\Phi$	Effective helix angle $= \tan^{-1} \left( \frac{V}{2\pi r n} \right)$
$T$	Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$		
$Q$	Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$		

#### 5. NUMERICAL RELATIONS

1 hp = 76.04 kg-m/s = 550 ft-lb/sec	1 lb = 0.4536 kg
1 metric horsepower = 0.9863 hp	1 kg = 2.2046 lb
1 mph = 0.4470 mps	1 mi = 1,609.35 m = 5,280 ft
1 mps = 2.2369 mph	1 m = 3.2808 ft